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THE INVERSE PROBLEM OF SUPERFAST CONTACT THEORY FOR AN ACCELERATED BODY WITH ORTHOTROPIC CONDUCTION

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One of the reasons restricting the use of conductors in railguns [1, 2] is the break of the current collector. This results from the current concentration in the neighborhood of the angle point A (Fig. 1), in the region with characteristic size $\Delta \cong \rho_2/(\mu_0 V)$, where ρ_2 is the specific resistance and V is the velocity of the body [3, 4].

In the present work, the method of reducing the current load in solid-state contact by use of a material with anisotropic (orthotropic) conductance is considered, and the current density distribution in a body consisting of many insulated conducting layers with specific resistance $\rho_1(x)$ is studied. The thickness of the layers satisfies the condition $\delta_1 \ll \Delta \ll d$, and the thickness of insulation $\delta_2 \ll d$ (d is the thickness of the body [armature]). Under such conditions, the distribution of the specific resistance in region 1 can be roughly considered continuous: $\rho_a(x) = \rho_1(x)[1 + \delta_2(x)/\delta_1(x)]$. The armature current density (region 2, Fig. 1) has only component j_y; therefore, the equation of diffusion of the field into the armature wall with induction $B = B_z$ in the system of coordinates of the armature can be written as

$$\frac{\partial B}{\partial t} = \frac{\partial E_x}{\partial y} + \frac{\partial}{\partial x}\left[\frac{\rho_a(x)}{\mu_0}\frac{\partial B}{\partial x}\right], \tag{1}$$

where E_x is the longitudinal component of the electric field strength; B and ρ_a do not depend on the coordinate y; therefore, $\partial E_x/\partial y$ is also a function of the x coordinate only and E_x is a linear function of y. In this case, $E_x(x, 0) = j_x(x, 0)\rho_2$, $E_x(x, -h) = -j_x\rho_2$ (j_x is the component of the rail current density. Thus, $E_x = j_x(x, 0)\rho_2(1 + 2y/h)$ and Eq. (1) finally takes the form

$$\frac{\partial B}{\partial t} = \rho_2 j_x(x, 0)\frac{2}{h} + \frac{\partial}{\partial x}\left(\frac{\rho_a(x)}{\mu_0}\frac{\partial B}{\partial x}\right). \tag{2}$$

The boundary conditions are $B(0) = B_e$ and $B(d) = 0$. The current density $j_x(x, 0) = -j_x(x, h)$ can be found by calculation of the field in the rails. The quasistationary condition $|\partial \mathbf{B}/\partial t| \ll |(\mathbf{V}\nabla\mathbf{B})|$ ($\mathbf{V}$ is the velocity of the body) is commonly used. Then, for the field in a rail with induction $B = B_z$ ($B_x = B_y = 0$), we have [3]

$$\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{1}{\Delta}\frac{\partial B}{\partial x} = 0 \tag{3}$$

[$\Delta = \rho/(\mu_0 V)$ is the thickness of the high-speed skin layer]. The numerical calculation of Eqs. (2) and (3) has shown that the use of a body with orthotropic conduction allows one to reduce heating considerably. For example, for the case $\rho_a = 10^{-6}\ \Omega\cdot$m, $V = 2\cdot 10^3$ m/sec, $B = 10$ T, the maximum Joule heat was $2\cdot 10^{12}$ W/m^3, which is nearly 8 times lower than that in a medium with isotropic conduction. The main attention here is devoted to the possibility of further reducing heating by means of forced current distribution

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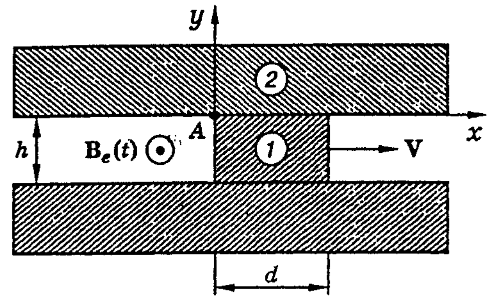

Fig. 1. Diagram of railgun: accelerated body (1), contact rails (2)

over the armature thickness, as in an analogous problem for stationary bodies [5–7], by the selection of the optimal dependence $\rho_a(x)$.

The Green function for Eq. (3) computed by Y. A. Dreizin and A. A. Kulakov has the form [8]

$$B^*(x_2, y_2) = \frac{1}{4\pi\Delta}\frac{y_2}{r_{12}}\exp\left[-(x_2 - x)/2\Delta\right] K_1(r_{12}/2\Delta), \tag{4}$$

where r_{12} is the distance between the points $1(x, 0)$ and $2(x_2, y_2)$; K_1 is the MacDonald function. Formula (4) allows one to calculate the induction in the rails with arbitrary distribution of induction on the boundary:

$$B(x_2) = \int_{-\infty}^{\infty} B^*(x_2, 0)\, B(x, 0)\, dx. \tag{5}$$

Further conversions (see Appendix) allow us to relate the tangential and normal components of the current density on the boundary:

$$j_x(x, 0) = -\frac{1}{4\pi\Delta}\int_{-\infty}^{\infty} \exp\left[-(x-\xi)/2\Delta\right]\left\{K_0\left[\,|(x-\xi)|/2\Delta\right] - K_1\left[(x-\xi)/2\Delta\right]\right\} j_n(\xi)\, d\xi \tag{6}$$

$[j_n(x) = -(1/\mu_0)\,\partial B/\partial x]$. Excluding $j_x(x, 0)$ by the use of Eq. (2), we obtain an integrodifferential equation for induction in the contact region $0 < x < d$:

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial x}\left[\frac{\rho_a(x)}{\mu_0}\frac{\partial B}{\partial x}\right] - \frac{\rho_2}{2\pi h \Delta \mu_0}\int_0^d \exp\left[-\frac{x-\xi}{2\Delta}\right]\left\{K_0\left[\frac{x-\xi}{2\Delta}\right] - K_1\left[\frac{x-\xi}{2\Delta}\right]\right\}\frac{\partial B(\xi)}{\partial \xi}\, d\xi = 0. \tag{7}$$

Instead of solving the direct problem [computation of the field in an accelerated body for a given function $\rho_a(x)$], it is expedient to formulate the inverse problem, that is to find $\rho_a(x) = \rho'_a(x)$ according to the given current distribution $j_y(x)$. The simplest example of such a problem is the case where the skin-effect in an accelerated body is negligible, and the term $\partial B/\partial t$ in Eq. (7) can be omitted. Then, from this equation, we obtain the following function for the specific resistance:

$$\rho'_a(x') = \rho_a(0) - \frac{\rho_2}{\pi\Delta h j(x')}\int_0^{x'}\int_0^{d'} T'(\lambda' - \xi')\, j(\xi')\, d\xi' d\lambda',$$

$$T'(\lambda' - \xi') = \exp\left[-(\lambda' - \xi')\right]\left\{K_0\left[|\lambda' - \xi'|\right] - K_1\left[(\lambda' - \xi')\right]\right\},$$

where $\rho_a(0)$ is the specific resistance on the undersurface of the armature; $x' = x/2\Delta$; $\xi' = \xi/2\Delta$; $\lambda' = \lambda/2\Delta$; λ is the integration parameter. The ratio $\rho'(x')/\rho_a(0)$ depends on two dimensionless parameters, $P_1 = \rho_2^2/[\mu_0\pi V h\rho_a(0)]$ and $P_2 = \rho_2 d/[h\rho_a(0)]$. The form of the function $\rho'_a(x')/\rho_a(0)$ is determined by the

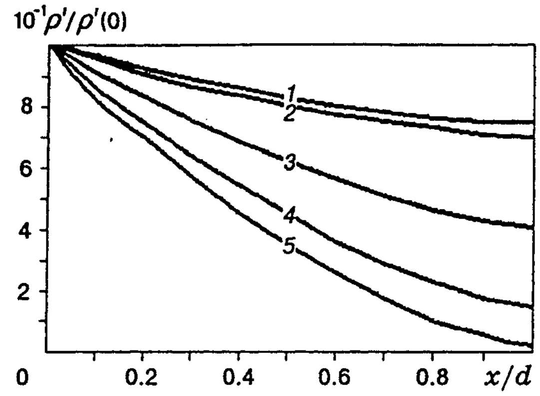

Fig. 2. Specific resistance distribution for $J(x) = \text{const}$: $P_1 = 0.00126$ and $P_2 = 0.1$ (1); $P_1 = 0.00126$ and $P_2 = 0.05$ (2); $P_1 = 0.00025$ and $P_2 = 0.1$ (3); $P_1 = 0.000126$ and $P_2 = 0.1$ (4); and $P_1 = 0.000025$ and $P_2 = 0.15$ (5)

relative current density distribution only. In particular, at $j_y(x) = \text{const}$, the solution has the form

$$\rho_a'(x') = \rho_a(0)\left\{1 - P_1 \int_0^{x'}\int_0^{d'} T'(\lambda' - \xi')\, d\xi' d\lambda'\right\}. \tag{8}$$

Figure 2 gives the results of using this formula for the following parameters of the system: $\rho_a(0) = 10^{-5}\ \Omega \cdot \mathrm{m}$ (which corresponds to the specific resistance of carbon plastic); the resistance of the steel rail is $\rho_2 \cong 10^{-6}\ \Omega \cdot \mathrm{m}$.

The results of the numerical solution of the problem in the range of parameters $P_1 = 10^{-4}$–10^{-3} and $P_2 = 0.005$–0.02 indicate that it is possible to ensure a given current distribution by using technically permissible distributions of specific resistance over the conducting layers of an anisotropic body. The function of the specific resistance, which decreases smoothly in the range from 10^{-5} to 10^{-6}–$10^{-7}\ \Omega \cdot \mathrm{m}$, corresponds to the modes considered. In the above particular case ($V = 2 \cdot 10^3$ m/sec, $B = 10$ T, $\rho_a = \rho_2 = 10^{-6}\ \Omega \cdot \mathrm{m}$), the transition from the uniform resistance distribution $\rho_a = \text{const}$ to $\rho_a'(x)$, which provides a constant current density (8), produces a 5-fold gain in heat dissipation. In this example, $q_{\max}$ can be reduced to $4 \cdot 10^{11}$ W/m^3.

The selection of the current density of the form $j_y = \text{const}$ is not optimal; the inner layers of the armature, where ρ_a is higher, are heated much more than the outer ones. It would be more proper to ensure a function $j_y(x)$ such that all the layers have the same heat loads, i.e., at each point with the x coordinate the condition of constant power [7] holds. Thus, in the cross section of the body we have

$$j_y^2(x)\, \rho_a(x) = \varphi^*(t), \tag{9}$$

where $\varphi^*(t)$ does not depend on x but can be a function of time.

We transform (3) into the equation for the current density $j_y(x) = -(1/\mu_0)\partial B/\partial x$. To this end, we differentiate it termwise with respect to x:

$$\mu_0 \frac{\partial j_y}{\partial t} = -\frac{2\rho_2}{h}\frac{\partial j_x(x,0)}{\partial x} + \frac{\partial^2}{\partial x^2}(\rho_a(x)\, j_y(x)).$$

From the condition (9), it follows that $j_y(x) = \varphi^*(t)/\rho_a^{1/2}(x)$. Then, using (6), we obtain the equation

$$\frac{\mu_0}{\rho_a^{1/2}(x)}\frac{\partial \varphi^*}{\partial t} = \varphi^*(t)\left\{\frac{\rho_2}{2\pi\Delta h}\int_0^d \frac{\partial}{\partial x} T[(x-\xi)/2\Delta]\frac{d\xi}{\rho_a^{1/2}(\xi)} + \frac{\partial^2}{\partial x^2}\rho_a^{1/2}(x)\right\},$$

which admits a solution under the exponential $\varphi^*(t)$, when $\varphi^*(t) = \varphi_0^*\exp(ct)$. In this case, we come to a

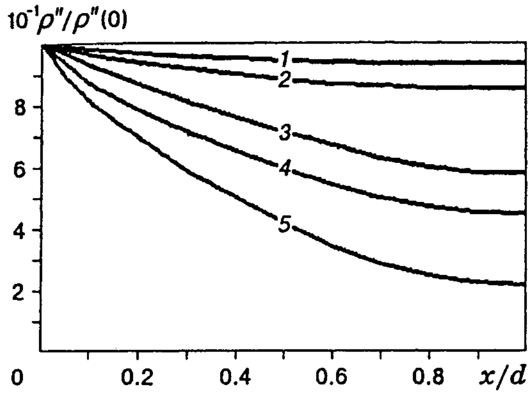

Fig. 3. Specific resistanse distribution for $dq/dt = \text{const}\,(x)$: $R_1 = 39.6$, $R_2 = 0.63$, and $R_3 = 1$ (1); $R_1 = 39.6$, $R_2 = 6.3$, and $R_3 = 0.25$ ($R_1 = 39.6$, $R_2 = 2.52$, and $R_3 = 0.5$) (2); $R_1 = 39.6$, $R_2 = 12.6$, and $R_3 = 0.25$ (3); $R_1 = 198$, $R_2 = 6.3$, and $R_3 = 1$ (4); and $R_1 = 158$, $R_2 = 6.3$, and $R_3 = 1$ ($R_1 = 39.6$, $R_2 = 6.3$, and $R_3 = 0.5$) (5)

nonlinear integrodifferential equation for the function $g = \rho_a^{-1/2}(x)$:

$$c\mu_0 g = \frac{\rho_2}{2\pi\Delta h}\int\limits_0^d \theta\left[(x-\xi)/2\Delta\right] g(\xi)\,d\xi + \frac{\partial^2}{\partial x^2}(1/g) \quad \left(\theta(t-x) = \frac{d}{dx}T(x-\xi)\right).$$

In the case of a constant field ($c = 0$), the equation is somewhat simplified:

$$\frac{d}{dx}\left(\rho_a^{1/2}(x)\right) + \frac{\rho_2}{2\pi h\Delta}\int\limits_0^d T(x-\xi)\,\frac{d\xi}{\rho_a^{1/2}(\xi)} = 0.$$

Introducing the notation

$$A(x',\xi') = \int\limits_0^{x'} \left(K_0(x'-\xi') - K_1(x'-\xi')\right)\exp(\xi'-x')\,dx',$$

we obtain the nonlinear integral equation

$$\frac{1}{g'(x)} - \frac{1}{g'(0)} = -\frac{1}{R_1}\int\limits_0^{R_2} A(x',\xi')\,g'(\xi')\,d\xi' \qquad (10)$$

$[g'(x) = \rho_a^{1/2}(x)]$, which can be solved by substitution of the integral by a finite sum. The form of the function $g'(x)$, and hence of $\rho_a(x)$, is determined by three dimensionless parameters: $R_1 = \pi\mu_0 h/2\rho_2$, $R_2 = \mu_0 V d/2\rho_2$, and $R_3 = (\rho_2/\rho_a(0))^{1/2}$. The reduced system of nonlinear algebraic equations corresponding to Eq. (10) was solved numerically by the Newton method. It is noteworthy that the iteration process does not converge at any combination of the dimensionless parameters $R_{1,2,3}$. Nevertheless, for some combinations of these parameters solutions were obtained and are presented in Fig. 3.

We can assess the efficiency of reduction of Joule heating under the condition that the heat release power is constant in x compared to the maximum speed of heat release at $j(x) = \text{const}$. In the first case,

$$j'(0) = \frac{B_0}{\mu_0 d}.$$

Therefore, the heat release power at the point $x = 0$ is

$$\dot{q}'(0) = \frac{\rho_a'(0)\, B_0^2}{\mu_0^2 d^2}. \qquad (11)$$

For the second mode $[\dot{q}'(x) = \text{const}]$,

$$\frac{d}{dx}\left(j''^2(x)\,\rho_a''(x)\right) = 0,$$

whence $j''(x) = j''(0)\,(\rho_a''(x)/\rho_a''(0))^{1/2}$. Integrating the relation $j = -(1/\mu_0)\partial B/\partial x$ and taking into account the latter equality, we obtain

$$B_0 = \mu_0 j_0'' \int_0^d (\rho_a''(x)/\rho_a''(0))^{1/2}\, dx,$$

whence

$$j_0'' = \frac{B_0}{\mu_0 d_{\text{eff}}} \quad \left(d_{\text{eff}} = \int_0^d (\rho_a''(x)/\rho_a''(0))^{1/2} dx\right).$$

Using the latter expression, for heating of the armature surface, we found

$$\dot{q}''(0) = \frac{B_0^2 \rho_a''(0)}{\mu_0^2 d_{\text{eff}}^2}. \qquad (12)$$

Comparing (11) and (12), we see that the transition from the distribution $\rho_a'(x)$ yielding $j = \text{const}$ to a distribution $\rho_a''(x)$ yielding $\dot{q}' = \text{const}$ will additionally reduce the heat release power by a factor of $(d_{\text{eff}}/d)^2$. In this case, this value is close to 4.

The results obtained allow us to optimize the distribution of the heat release power in the region of the current collector. However, the actual temperature distribution is also determined by the process of heat transfer from the heated armature to the relatively cold rail [9]. Therefore, the ultimate answer concerning the efficiency of the proposed optimization can be given only by solving the heat conduction equation together with the field equations.

Appendix. The magnetic field in the rail can be expressed in terms of the field on the boundary and the fundamental solution (4) according to the formula

$$B(x_2, y_2) = \int_{-\infty}^{+\infty} B^*(x_2, y_2)\, B(x, 0)\, dx.$$

Thus, in the case of point contact, we have

$$B(x_2, y_2) = \frac{B_0 y_2}{4\pi\Delta} \int_{-\infty}^{0} \frac{1}{r_{12}} \exp\left(-\frac{x_2 - x_1}{2\Delta}\right) K_1\left(\frac{r_{12}}{2\Delta}\right) dx_1 \quad \left(r_{12} = \sqrt{y_2^2 + (x_2 - x_1)^2}\right).$$

Using the integral representation of the function K_1

$$\frac{y_2/(2\Delta)}{\sqrt{y_2^2 + (x_2 - x_1)^2}} K_1\left(\frac{1}{2\Delta}\sqrt{y_2^2 + (x_2 - x_1)^2}\right) = \int_0^\infty \exp(-y_2\sqrt{t^2 + a^2}) \cos(|x_2 - x_1|t)\, dt,$$

and then integrating with respect to the variable x_1, we obtain the following expression for the magnetic field at an arbitrary point subject to boundary conditions defined above:

$$B(x_2, y_2) = \frac{B_0 \exp(-\alpha x_2)}{2\pi} \int_0^\infty \exp\left(-y_2\sqrt{t^2 + a^2}\right) \left(\frac{\alpha\cos(x_2 t)}{\alpha^2 + t^2} - \frac{t\sin(x_2 t)}{\alpha^2 + t^2}\right) dt.$$

Here $\alpha = 1/(2\Delta)$; t is the variable connected with the integral representation of the function K_1.

Hence, the expression for the x component of the current density can be obtained.

$$j_x(x_2,y_2)=\frac{1}{\mu_0}\frac{\partial B(x_2,y_2)}{\partial y_2}=-\frac{B_0\exp(-\alpha x_2)}{2\pi\mu_0}\left(\alpha K_0(\alpha\sqrt{x_2^2+y_2^2})-\alpha K_1(\alpha\sqrt{x_2^2+y_2^2})\frac{x_2}{\sqrt{x_2^2+y_2^2}}\right). \tag{A.1}$$

It is of prime interest to determine the value of the current density on the boundary of the regions (in the contact zone), i.e.,

$$j_x(x_2,0)=-\frac{B_0\exp(-\alpha x_2)\alpha}{2\pi\mu_0}\Big(K_0(\alpha|x_2|)-K_1(\alpha|x_2|)\operatorname{sign}(x_2)\Big).$$

To assess these values, we can employ asymptotic dependences for the MacDonald functions and get an expression for the current density in the region ahead of ($\alpha x_2\to\infty$) and behind ($\alpha x_2\to-\infty$) the contact zone. As a result, in the region ahead of the contact [$\operatorname{sign}(x_2)>0$], we have

$$j_x(x_2,0)\to\frac{B_0\exp(-\alpha x_2)}{4\mu_0\sqrt{\pi x_2}}\frac{\sqrt{\Delta}}{x_2},$$

while behind the contact

$$j_x(x_2,0)\to\frac{B_0}{2\mu_0\sqrt{\pi|x_2|\Delta}}.$$

An analysis of the relations obtained suggests that there are pronounced peculiarites as $x_2\to0$, i.e., in the point contact zone.

We consider the influence of the transition from point contact to an arbitrary distribution of the normal component of the current density on the boundary.

An expression for the x component of the current density in this case can be derived if we introduce $di(x_1)=j_n(x_1)\,dx_1$ in place of the coefficient $i'=B_0/\mu_0$ in (A.1), replace x_2 by x_2-x_1, and perform integration. As a result of the above operations, expression (5) is transformed into the relation

$$j_x(x_2,0)=-\frac{1}{4\pi\Delta}\int\limits_{-\infty}^{+\infty}\exp(-\alpha(x_2-x_1))\,(K_0(\alpha|x_2-x_1|)-K_1(\alpha|x_2-x_1|)\operatorname{sign}(x_2-x_1))\,j_n(x_1)\,dx_1.$$

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